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NUMERICAL SOLUTION OF THE INTERNAL INVERSE PROBLEM FOR AN ANISOTROPIC SYSTEM WITH SOURCES

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The authors have developed and implemented an approach to the problem of modeling the temperature fields of anisotropic systems with sources on the basis of the equivalence principle.

Introduction. Because of complicated and tedious solution, mathematical models of heat transfer processes in three-dimensional anisotropic systems with sources are not easily treated in a numerical experiment, even in the rare cases where the form, parameters of the model, and other conditions of uniqueness are known. In most cases components of the model of the system (characteristics of the interaction, thermophysical properties of the components, and other subsystems) are known approximately and the structure of the system is irregular. Because of this, simplified models of heat transfer have to be used.

Below, we will consider a radioelectronic unit (REU) as an example of these systems. The procedure of calculating the temperature fields of REU is known to employ methods of step-by-step modeling, i.e., successive use of thermal and mathematical models corresponding to the increasing degree of structural details of the system [1]. In the design step, when the final structure of the system is not known, the problem of determining the equivalent averaged thermophysical properties of the system is very important. Details of the structure and properties of particular components in similar designs have only a slight effect on the values of equivalent characteristics of these systems [2]. Therefore, the values obtained can be used for a rather wide class of systems with similar structure and properties [3, 4].

The equivalent characteristics of the systems considered are calculated using methods for solution of inverse problems [5]. The conditions of equivalence of specific characteristics of the model and the real system can be different [6-8]. The use of the equivalence principle from phenomenological heat conduction theory allows the equivalence conditions to be formulated as a coincidence of the model and real temperatures at some prescribed points of the system at preset moments of time with a required accuracy [8]. In this case a simplified model (bodies of a simple shape, homogeneous structure, etc.) can be used for solution of the direct heat transfer problem.

Mathematical Model. As a simplified heat transfer model of the radioelectronic unit we used a linear unsteady-state heat conduction equation for a cylinder of actual dimensions (r = 80 mm, z = 160 mm) with an isotropic or orthotropic structure with boundary conditions of the third kind (α = 10 W/(m·K)) over the external surface and initial conditions:

$$c_v \frac{\partial t}{\partial \tau} = \lambda \frac{\partial^2 t}{\partial r^2} + \frac{\lambda}{r} \frac{\partial t}{\partial r} + \lambda \frac{\partial^2 t}{\partial z^2} + \frac{\lambda}{r^2} \frac{\partial^2 t}{\partial \varphi^2} + q(r, \varphi, z); \quad (1)$$

$$c_v \frac{\partial t}{\partial \tau} = \lambda_r \frac{\partial^2 t}{\partial r^2} + \frac{\lambda r}{r} \frac{\partial t}{\partial r} + \lambda_z \frac{\partial^2 t}{\partial z^2} + \frac{\lambda r}{r^2} \frac{\partial^2 t}{\partial \varphi^2} + q(r, \varphi, z); \quad (2)$$

$$\alpha(r, \varphi, z, t_s)(t_{\text{med}} - t_s) = -\lambda \frac{\partial t}{\partial n}\Big|_s; \quad (3)$$

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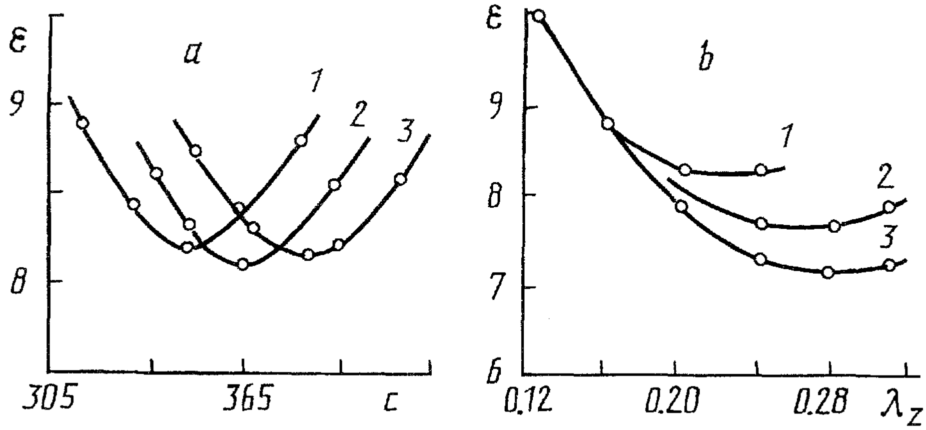

Fig. 1. The discrepancy in inverse problems without sources; a) anisotropy: 1) $\lambda = 0.18$; 2) 0.20; 3) 0.22 W/(m·K); b) orthotropy: 1) $\lambda_{r\varphi} = 0.2$; 2) 0.12; 3) 0.04 W/(m·K). ε, K; c, J/(kg·K); λ_z, W/(m·K).

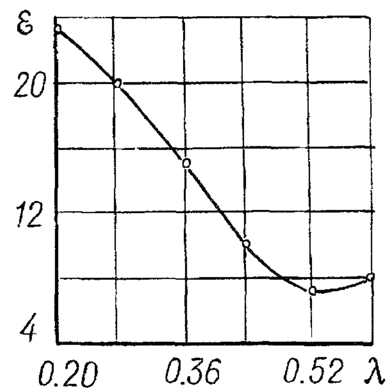

Fig. 2. The discrepancy in a problem with sources: isotropy. λ, W/(m·K).

$$t(r, \varphi, z, 0) = \text{const}. \tag{4}$$

The form of the orthotropy ($\lambda_z \neq \lambda_r = \lambda_\varphi$) was determined by the transverse position of the boards with heating elements in the unit. Numerical solution of model (1)-(4) was carried out by an implicit finite-difference scheme on an ES-1060 computer; the total number of nodes is 2112, the number of nodes along the axes r, φ, z is 8, 24, and 11, respectively. The number of sources is 261.

Problem without Sources. The inverse problem without sources was solved in a two-dimensional formulation for half of an axial section of the cylindrical volume of the unit in the coordinates r, z. The calculation results were compared with the experimental temperature field of the unit, heated in a gaseous medium with a constant temperature under natural convection. The temperature distribution was measured by thermocouples fixed at random positions in the unit (on the boards, in the air, on the casing).

The largest absolute difference of the model and experimental temperatures at all measuring points at all times was used as the discrepancy. In solution of the inverse isotropic problem the smallest discrepancy at all points of the system was $(8\pm0.1)\%$ and here $\lambda = 0.2$ W/(m·K) and $c = 360$ J/(kg·K). For the orthotropic problem with the same values, the discrepancy was a little smaller, amounting to $(7\pm0.1)\%$, and here $\lambda_r = 0.04$ W/(m·K) and $\lambda_z = 0.28$ W/(m·K) (Fig. 1).

Problems with Sources. It can be shown that the equivalent thermal conductivities of anisotropic systems should be different with different boundary conditions and different powers and locations of surface and volume heat sources. This was shown for steady-state temperature conditions, in particular, in [9]. Numerical calculations confirm these quantitative conclusions.

Results of solution of the direct problem of calculating the temperature field of the unit heated by heat sources were compared with experimental data (of a different experiment) on heating by sources under operating conditions. With sources, the equivalent thermal conductivities turned out to be higher, other things being equal. They were 0.46 W/(m·K) instead of 0.2 W/(m·K) (Fig. 2). For this problem the maximum discrepancy was smaller for the isotropic model than for the orthotropic model, while in solution of the inverse problem without sources, the discrepancy was smaller for the orthotropic model.

Conclusion. In solution of inverse heat transfer problems for an anisotropic system with sources it is possible to determine its equivalent thermophysical properties, corresponding to a simplified mathematical model of the process and experimental temperature fields, both for heating in a medium with constant temperature and for heating by intrinsic sources. The resultant values can be used in solution of direct problems of determining the temperature field in a known range of properties of the components, the character of the structure, and changes in the initial boundary conditions for a complicated mathematical model of the systems studied. When obtaining and using the equivalent properties, one should remember that the equivalence conditions must not violate the conditions of equality of the supplied and removed energies in the initial complicated and simplified thermal models of the process.

NOTATION

c, specific heat; c_V, volumetric heat capacity; q, heat flux density; r, φ, z, cylindrical coordinates; t, temperature; α, heat transfer coefficient; ε, discrepancy; λ, thermal conductivity; τ, time. Subscripts: r, φ, z, coordinates; s, surface; med, medium.

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